Automatic shape optimization of the first step labyrinth seal of a geothermal steam turbine

Optimización automática de la forma del sello laberintico de primer paso de una turbina de vapor geotérmica

Miguel Ángel Tamayo Soto[1], Ángel Cerriteño Sánchez[1], Heriberto Arias Rojas[2], Francisco Javier Domínguez Mota[2], Nicolás David Herrera Sandoval[3] Sergio Ricardo Galván González[1*]

Highlights

- The shape modification of a labyrinth seal with realistic manufacturing possibilities could increase the performance of a geothermal steam.
- The design of a power shell program is able to couple multidisciplinary software to a Genetic Algorithm to found the optimal shape of the labyrinth seal.
- There exists an exact shape of the labyrinth seal which could to make the energy transfer in each turbine step more efficient.

Innovaciencia ISSN: 2346-075X

E- ISSN: 2346-075X
Innovaciencia 2025; 13(1); e4996
http://dx.doi.org/10.15649/2346075X.4996

ORIGINAL RESEARCH AND INNOVATION ARTICLE

How to cite this article:
Tamayo Soto MA, Cerriteño Sánchez A, Arias Rojas H, Domínguez Mota FJ, Herrera Sandoval ND, Galván González SR. Automatic shape optimization of the first step labyrinth seal of a geothermal steam turbine. Innovaciencia. 2025;13(1):e4996. http://dx.doi.org/10.15649/2346075X.4996

Received: February 22, 2025
Accepted: June 19, 2025
Published: August 08, 2025

Keywords:

Labyrinth Seal; Steam Turbine; Numerical Optimization; Genetic Algorithms.

Palabras clave:

Sello Laberíntico; Turbina de Vapor; Optimización Numérica; Algoritmos Genéticos.

ABSTRACT

Introduction. The design of straight labyrinth seals for a steam turbine has had little progress concerning their geometry. As an auxiliary system, its current shape causes a significant loss of efficiency in this machine. **Objective.** This study aims to evaluate the use of an automatic optimization process to define a more suitable seal profile that increases the pressure ratio at the first stage of a steam turbine installed in a geothermal field in México. **Materials and Methods.** Multidisciplinary software, controlled by a PowerShell script, were coupled to optimize the seal shape in response to the flow and pressure field given by a computationally cheap CFD model of the first step of the labyrinth seal. **Results and Discussion.** By parameterizing the seal geometry, the leading edge was automatically manipulated with the numerical algorithm, obtaining an angle tendency towards 45°. This new leading edge decreased the outlet pressure of the seal system by 1.2%, which, for a total of 5 stages in the turbine with the same seals, would result in a 6% pressure reduction. **Conclusions.** It is inferred that this practical methodology could easily update the seal design during the turbine re-powering operations.

RESUMEN

Introducción. El diseño de los sellos de laberinto rectos en las turbinas de vapor ha tenido pocos avances en cuanto a su geometría. Como sistema auxiliar, su forma actual genera una pérdida significativa de eficiencia en estas máquinas. **Objetivo.** Este estudio tiene como objetivo evaluar el uso de un proceso de optimización automática para definir un perfil de sello más adecuado que aumente la relación de presiones en la primera etapa de una turbina de vapor instalada en un campo geotérmico en México. **Materiales y métodos.** Se acoplaron diferentes programas multidisciplinarios, controlados mediante un script en PowerShell, para optimizar la geometría del sello en respuesta al campo de flujo y presión obtenido a partir de un modelo CFD computacionalmente económico de la primera etapa del sello de laberinto. **Resultados y discusión.** Al parametrizar la geometría del sello, el borde de ataque fue manipulado automáticamente mediante el algoritmo numérico, obteniendo una tendencia angular cercana a los 45°. Este nuevo borde de ataque redujo la presión de salida del sistema de sellado en un 1,2%, lo que, considerando un total de cinco etapas en la turbina con sellos similares, resultaría en una disminución de presión del 6%. **Conclusiones.** Se infiere que esta metodología práctica podría actualizar fácilmente el diseño de los sellos durante las operaciones de repotenciación de turbinas.

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[1] Universidad Michoacana de San Nicolás de Hidalgo, Facultad de ingeniería Mecánica, Morelia, México.
[*] Corresponding author: sergio.galvan@umich.mx
[2] Universidad Michoacana de San Nicolás de Hidalgo, Facultad de Ciencias Físico-Matemáticas, Morelia, México.
[3] Tecnológico Nacional de México, Campus Morelia, Morelia, México.

INTRODUCTION

Steam turbines are fluid flow machines that move generators in the power industry Worldwide, more than 50% of the electrical energy is generated by steam turbines. Even with the alternative energies, steam turbines have continued to be an essential source of electrical generation. To obtain satisfactory overall performance, this type of turbine requires a significant pressure difference between the inlet and outlet sections to produce the steam expansion in different stages. In the steam turbine, the high-pressure, high-temperature steam from nozzle flows in an annular space to the stationary blades and is directed tangentially against the rotating blade (1). Several years ago, at the beginning of the steam turbine development, their efficiency was 45% to 56%. Modern high-tech software and innovative materials have helped to increase it, ranging from 72% to 78%, (2).

However, the constant loss of working fluid among different turbine components has been the main obstacle to achieving maximum efficiency. The distribution of typical efficiency losses in one steam turbine stage is presented in (**Table 1**). Among the leakage losses, the most significant one is tip leakage, which accounts for 22%, corresponding to approximately one-third of the total stage efficiency loss. This loss is caused by a small gap designed to allow fluid to pass at the upper end of the moving blades in the expansion stage. The tip leakage losses during steam entry into the turbine is showed in (**Figure 1**).

Table 1. Typical sources of steam turbine stage efficiency loss(3)

Loss	Percent
Rotation	3%
Root Leakage	4%
Carryover	4%
Shaft Packing Leakage	7%
Bucket Profile	15%
Nozzle Profile	15%
Bucket Secondary	15%
Nozzle Secondary	15%
Tip Leakage	22%

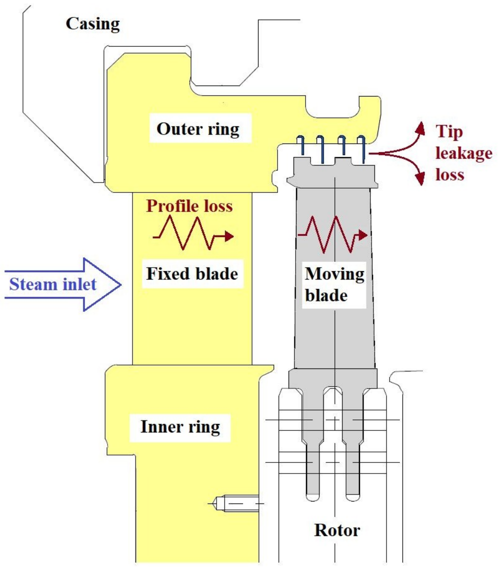


Figure 1. Location of the tip leakage loss in an expansion stage of a steam turbine.

To avoid the efficiency loss at the tip leakage stage, a sealing system that works according to the principle of the pressure ratio was implemented by Yu *et al.*(4). As the fluid passes through the geometries of the seals, it fills the cavities, forming swirls inside them, causing a decrease in the leakage velocity. Thus, the pressure energy is converted into eddies and heat energy.

Different companies have replaced the seal design of conventional labyrinths with more advanced turbomachinery sealing technology, because they are particularly suitable for high speed, high-pressure gas sealing applications due to the simple, compact, and robust construction (5). Nonetheless, the straight labyrinth seals are the most common in sealing systems that seek to maximize steam choking at the turbine blade tip. This is because they are easy to manufacture, inexpensive, and simple in their arrangement, offering a possibility for obstructing flow from a high to a low-pressure region and functioning as replacement parts in the maintenance of the steam turbines (6).

Thus, some researchers have reported procedures to maximize the labyrinth seal performance by modifying its shape, authors have presented stylized shapes at the tip of the seals (7), or an intermediate extraction in the steam circulation system (8), which would diagnose its state. After a significant maintenance procedure on a 25 MW turbine, Darshan (9) noticed that a crush in the seal increased its efficiency, which could have transformed the sealing element into the ideal geometry to increase dissipation and operate closer the maximum efficiency point. The steam turbine installed in two of the five geothermal fields in Mexico and used under commercial operation, the seal system presented a slight forward deformation, improving the performance (10).

Thus, the previous studies have demonstrated that a practical geometry has been vital to increase the efficiency of the turbine. It is assumed that such deformation modified the flow trajectory, creating vortices in the cavities of the chambers. These vorticities, in turn, provoked a blockage in the clearances, increasing the pressure ratio.

However, to find the ideal shape that modifies the flow trajectory along the chamber cavities of the seal stage to reach the maximum static pressure difference turns this work into an optimization shape design.

Moreover, the number and combination of the design parameters to define the seal geometry in response to a flow trajectory modification could end in an unpractical process Additionally, the limitation of the current commercial technology to manufacture this type of piece has always been a common factor.

In order to avoid a non-smooth optimization problem, which yields an unpractical process to determine the optimal shape, this paper proposes to parametrize the geometry for straight seals commonly used in steam turbines in response to the flow evaluation.

Recent research has introduced new numerical tools based on Computational Fluid Dynamics (CFD) and optimization algorithms to evaluate the performance of hydraulic structures and machinery. Among them, Galván *et al.* (11) showed that coupling optimization and CFD algorithms systematically improved the geometric design of a turbomachinery component due to the flow information obtained from CFD predictions. Thus, taking this same principle, an automatic process was designed to find the geometric profile in response to the flow behaviour in a research space limited by geometric restriction.

However, since the high complexity of the real flow in the chamber of the seal requires an important number of computational resources, it is necessary now to abandon the idea of an exact evaluation of the objective function. Thus, reducing the model complexity of the CFD configuration to speed-up each evaluation, while keeping all (or most) of the needed coupling processes describing the important physics, could be useful to maintain the optimization process in a computationally affordable way, which could be useful for industrial applications.

MATERIALS AND METHODS

The steam channel of the reference turbine has seven blade-stages, as is shown in (**Figure 2**), and only the first five ones use straight seals. The last two stages of this turbine work under low pressure at the steam outlet and the seal is not required. This study will focus on the analysis of the first stage where the Interest Domain (ID) and channel characteristics are remarked.

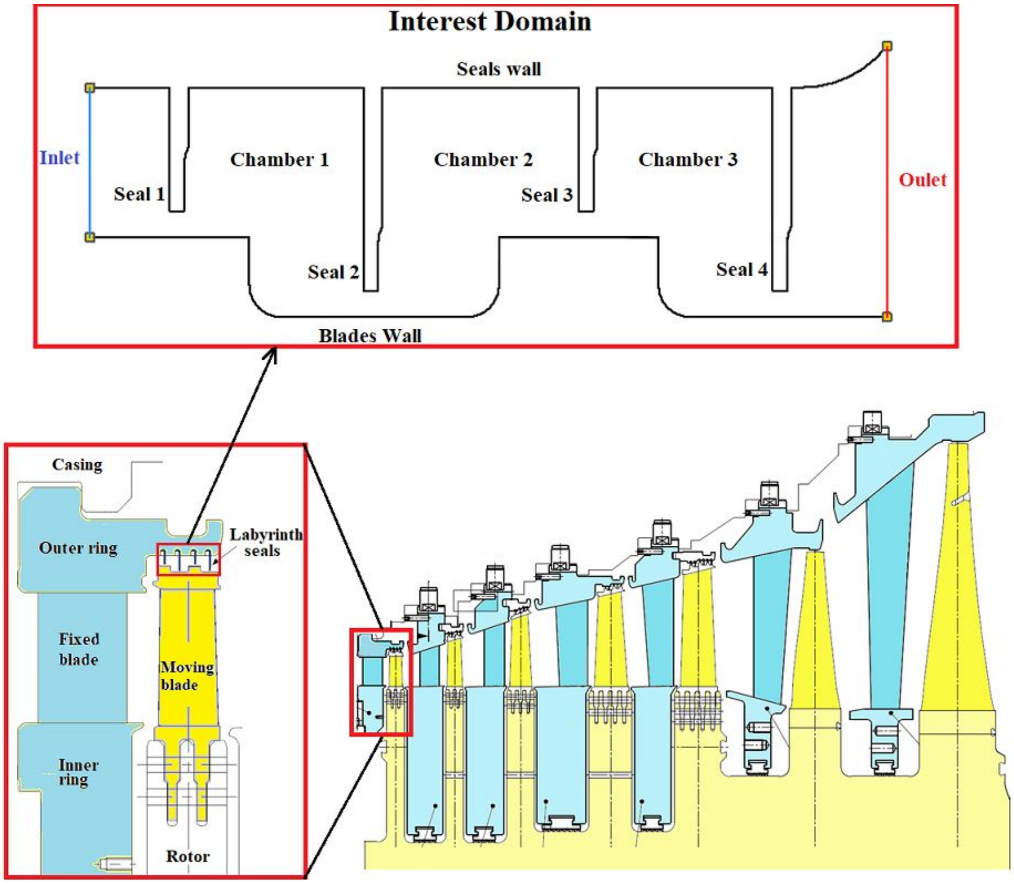


Figure 2. Interest Domain of the stepped labyrinth seal system in stage one of the reference turbine.

Optimization design process

The automatic optimization process designed in the PowerShell is described in (**Figure 3**). By the Flow diagram using the following steps:

1. The optimization algorithm assigns an initial value to each design variable using a data file.
2. The design parameters of the seal profile are read.
3. The geometry of the seal is built.
4. The computational domain, including the topology, mesh, and boundaries are created.
5. The CFD simulation is executed.
6. The static the static pressure at the outlet is defined.
7. The performance quantities are evaluated.
8. The minimization of the objective function is evaluated.
9. If the condition is completed, the optimized geometric parameters are obtained. Otherwise, the process is sent back to the step 1.

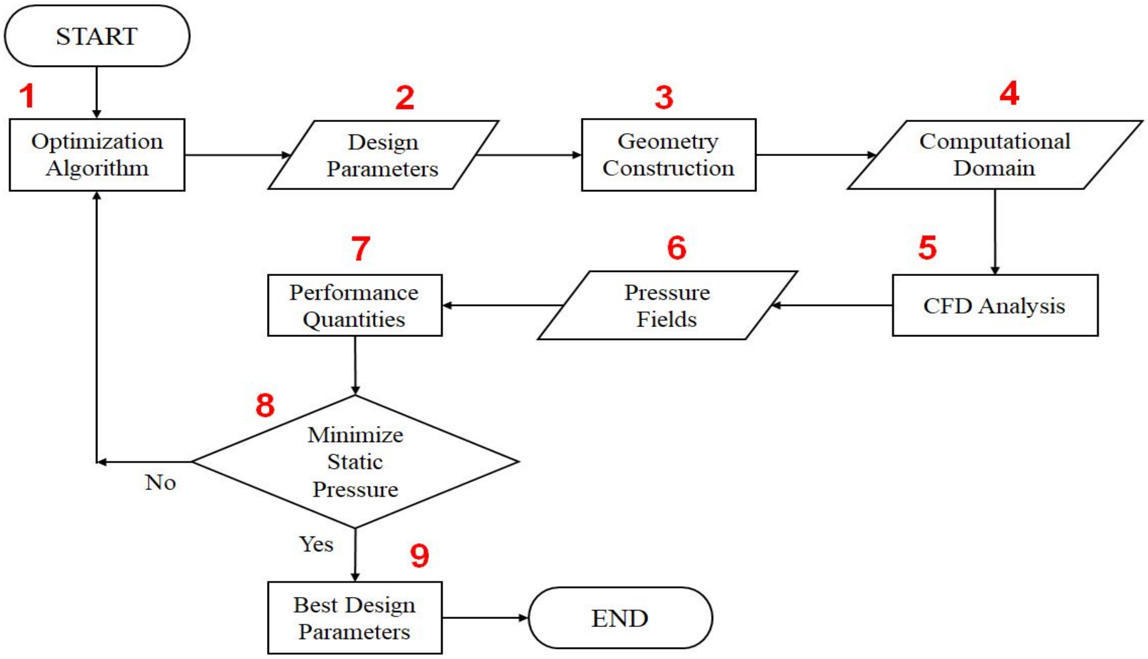


Figure 3. Flow diagram executed by the PowerShell to reach the shape optimization of the seal.

As presented by Galván *et al.* (11), Herrera *et al.* (12) and Rulik *et al.* (13), the original seals were manipulated by an optimization algorithm coupled to a CFD and Computer Aided Design (CAD) software, looking for the ideal shape. The design of a PowerShell script to couple Isight, MATLAB, Gambit, and Fluent software transformed the seal shape design described by (**Figure 5**), into an automatic optimization process (**Figure 4**)

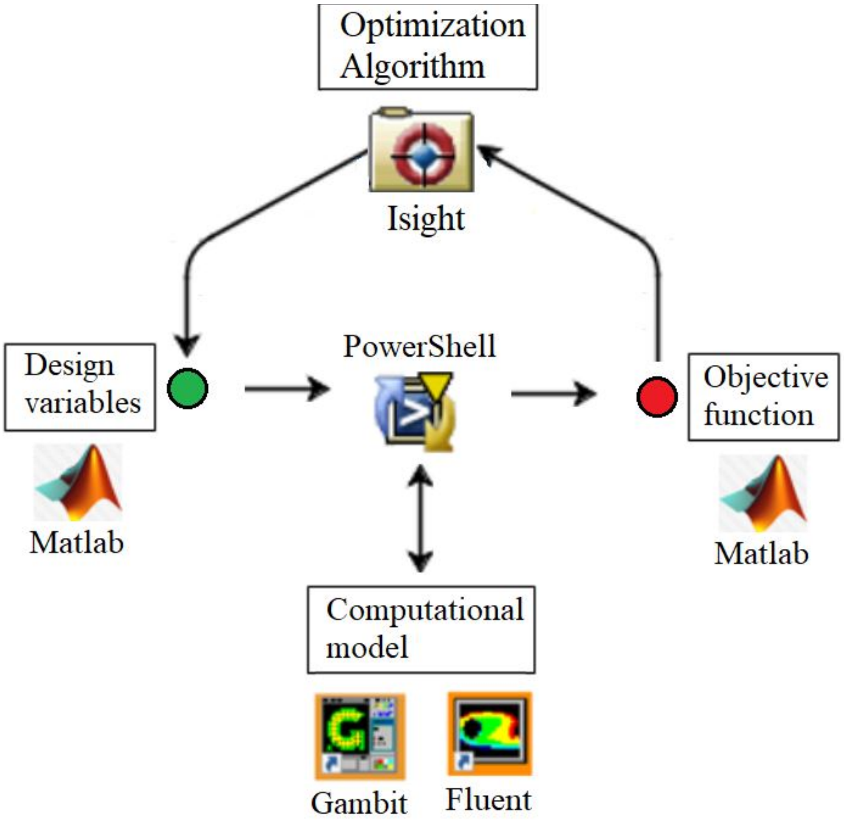


Figure 4. Coupling of the software used in the automatic optimization of the labyrinth seal design.

However, the optimization strategies that should guarantee the reliability, flexibility and computationally affordable automatic process are given by the next requirements:

- The minimum input geometric parameters.
- The rapid response low-fidelity CFD model.
- The distributed optimization algorithm.
- The single-value objective function.

Geometry parameterization

Clearance tolerances are typically 1.0 mm or less for High-Pressure (HP) turbines, increasing to more than 2.0 mm for a sealing system in Low-Pressure (LP) turbine stages (4). Based on the coordinates that will be defined for the parameterization, two circles were created to form the geometry of the leading edge of the seal automatically. The position and curvature of the stamp can be varied, and the slope Equation (1) points to the line to locates the second point that defines the path of the two concentric circles, to form the leading edge of the labyrinth seal.

$$y_2 - y_1 = m(x_2 - x_1) \tag{1}$$

According to the steam turbine's original design, there are some established geometrical restrictions for the problem statement:

a) the clearance between the rotor disc (g = 1 mm) and
b) the seal thickness (t = 2 mm).

Thus, x-y coordinates were defined to draw concentric circles, as shown in (**Figure 5**), where the shaded area represents the new seal geometry.

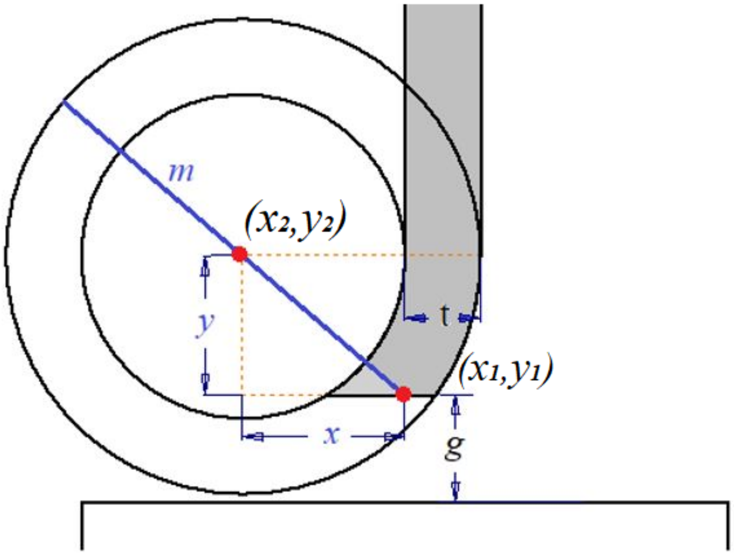


Figure 5. x-y coordinates of the leading edge of the new seal.

The design value of the slope m and the coordinate x, which define the position of the leading edge, are presented in (**Table 2**). The design range of the variables must maintain a realistic geometry configuration.

Table 2. Design parameters range for automatic construction of seal geometry.

Parameter name	Minimum (mm)	Maximum (mm)
x	-0.001	-4.5
m	-0.8	-0.89

x: Horizontal distance for the automatic modification of the seal's curvature, based on the slope. *m*: Slope or inclination derived from two defined points to form the seal geometry.

The geometry evolution of the seals' leading edge, using different values of the geometrical parameter in the *x* direction, is shown in (**Figure 6**). The adjustment of the seal geometry should be made automatically by modifying the *x* variable.

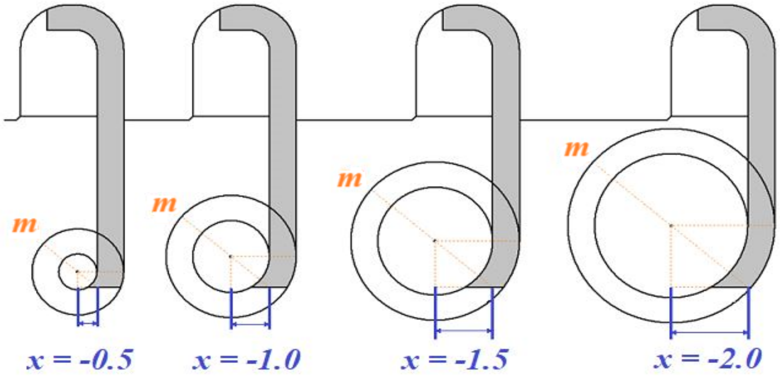


Figure 6. Geometric evolution of the inclined stamp for the seal based on *x* coordinate.

Computational model

Gambit software was used to create the domain geometry od the (**Figure 2**) and to generate the structured mesh to control the number of elements, their dimensions and the quality. Thus, a Multiblock mesh, (**Figure 9**), with 8332 elements was the reference mesh that provided suitable conditions (rapid construction of fewer elements and easy manipulation) for adaptation to the automatic optimization process.

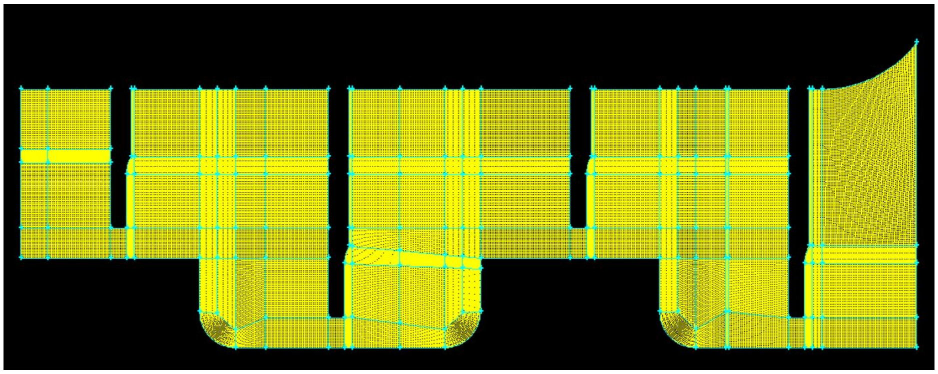


Figure 7. Multiblock meshes for a sealing system with straight seals.

To qualitatively validate the CFD model, the CAD step was printed on a scale 10:1 to allow the visualize flow trajectory visualization along it. The ID was 3D printed in Polylactic Acid. Smoke was injected with a tracer to observe the flow behaviour. (**Figure 8**)

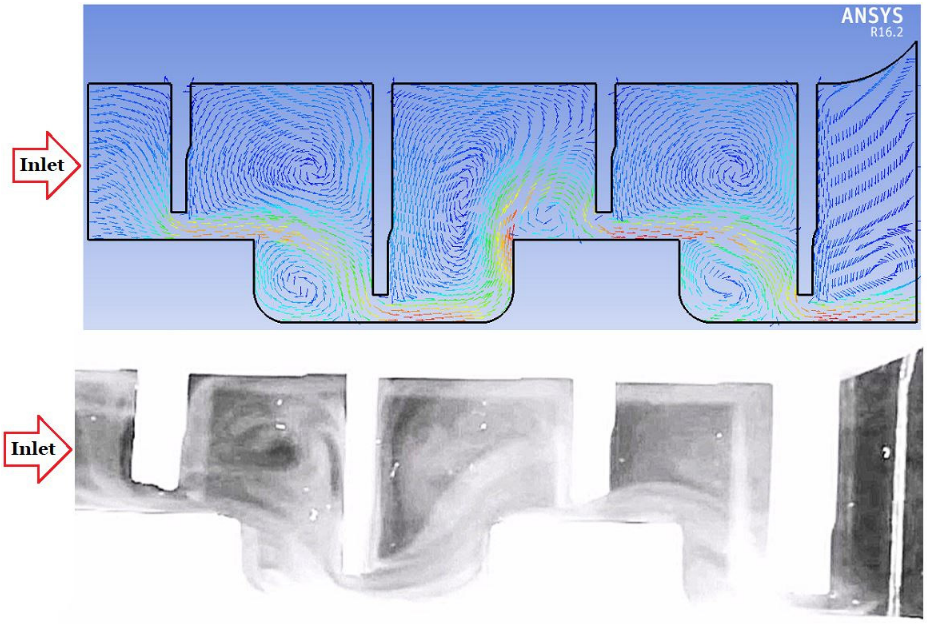


Figure 8. Comparison of the flow trajectory simulation in a scaled physical model of straight seal systems.

The mesh quality, when the geometry of the seal is extremely modified ($x = -2.0$, see (**Figure 6**)), is presented in (**Figure 9**). The mesh reaches a skew size of 0.53 in the 17 most deformed elements.

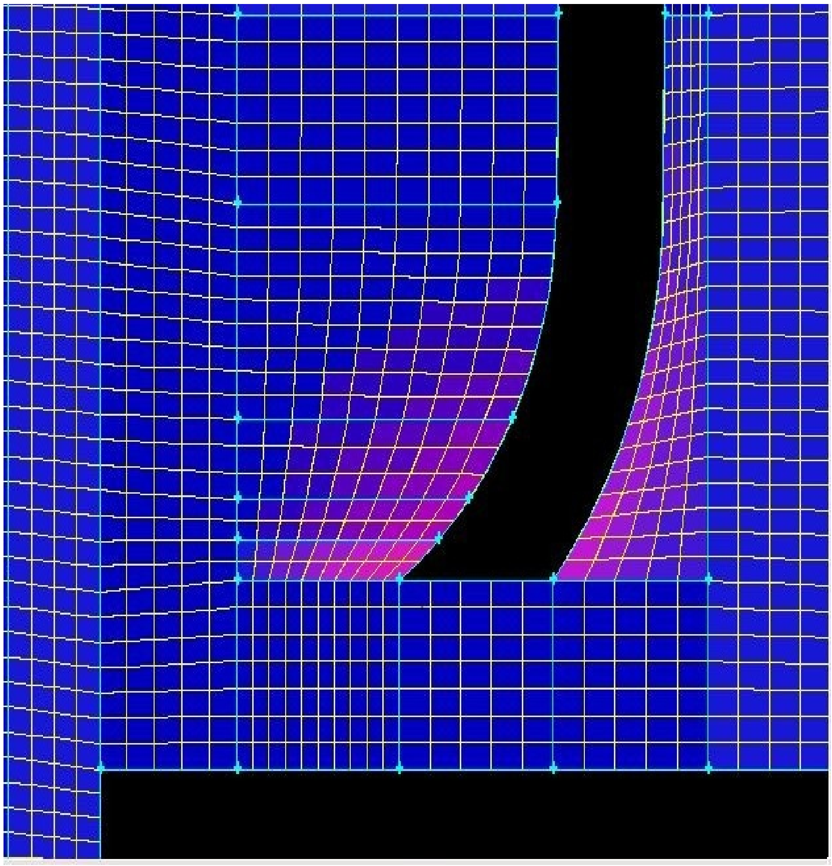


Figure 9. Highest deformation in the mesh elements (red), provoked by the maximum modification of the seal geometry.

A well-defined and computationally cheap 2D CFD model can help to analyse many design configurations in a relatively short time, Kim and Cha (14) and Xi and Rhode (15) justify the case of study as axial symmetric, 2D axi-symmetric models are adequate to accurately predict the seal leakage flow rate using a coarse grid (16), the reduced computational model considers compressible flow, a conservative form of the Navier-Stokes equations, and along with the k - ∈ model and high Reynolds number used by Kim and Cha (14), Mohammadi and Pironneau (17) and Zhao and Wang (18), other turbulence models still show discrepancies compared to measurements (19), for this kind of analysis, a 3D computational model is not a practical option. It has been verified that the difference between using 2D and 3D models is only within a 0.03% margin (20).

Inlet boundary Conditions

Equation (2) can be used in turbomachinery seals where the steam is a perfect gas, considering state equation.

$$P = \gamma \rho R T \tag{2}$$

Where P is the pressure, γ is the compressibility factor, is the density, R is the universal gas constant, and T the temperature.

The reference operating parameters of the geothermal turbine, as well as the steam conditions applied to the inlet boundary, are presented in (**Table 3**). The uniform distribution of steam pressure injection along the inlet and outlet areas of the system suggests the axis-symmetrical condition used by Chakravarthy and Srikanth. (21). This condition justifies the two-dimensional analysis along the labyrinth channel.

Table 3. Turbine inlet steam conditions.

Parameter name	Condition	Value
Steam quality	Saturated steam	1
Temperature (°C)	Maximum 172.45	170.42
Absolute pressure (Bar)	Maximum 8.4	8.4

Condition: It is the physical measurement data of the turbine inlet steam. Value: It is the value considered for the initiation of the optimization process; based on this value, the steam behaviour within the seal chamber will be observed in the simulation.

Optimization algorithm

Genetic Algorithms (GA) have been an adaptive method for solving shape design optimization problems. The power of GAs comes from their robust technique in solving various problems from different areas. The multi-island is a sequential GA with regular exchanges of subpopulations between islands according to the pre-established number of generations and some defined parameters (22).

The division of individuals into subpopulations that evolve in a parallel and independent manner through migrations, which allow the exchange of genetic material, is illustrated in (**Figure 10**). Moreover, elitism guarantees that the best genetic material is preserved across generations by carrying over the best individual from the previous generation without alteration.

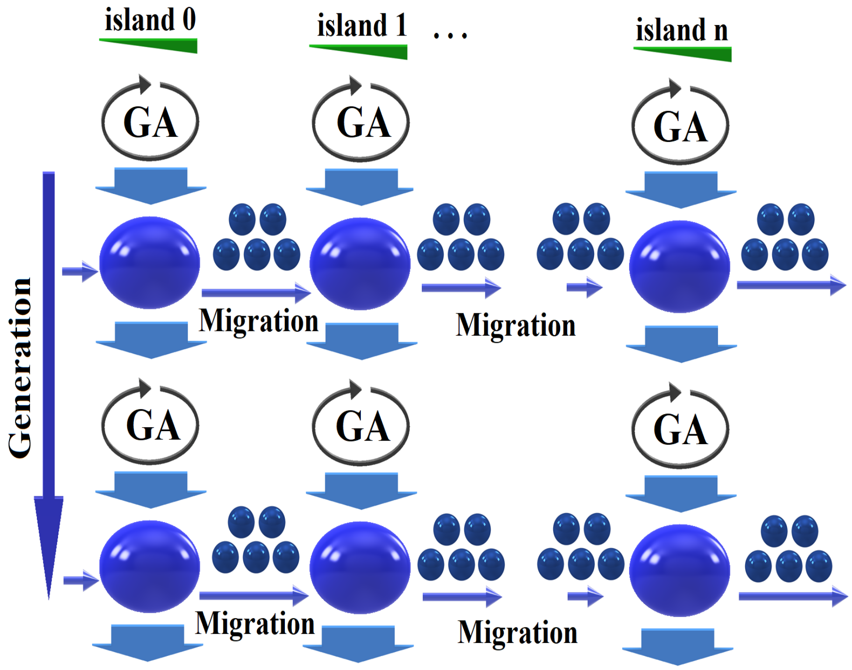


Figure 10. Exchange of information of the Evolutionary Algorithm among islands and towards the next generation.

The optimization algorithm intended to obtain the minimum value in the outlet static pressure which was implemented in a parallel and distributed computing cluster.

For the distribution, three individuals (designs) were grouped in three islands using three generations, obtaining 23 evaluations that ran in parallel until the end of the process. Each calculation nodes parallelized the computational domain of each new seal design to accelerate the process.

The objective function

Since the main characteristic of the seal is to modify the pressure loss along the domain(23), it is expected that the geometric inclination of the seal leading edge can modify the flow trajectory inside each chamber. Thus, the number of vortices in the pressure chambers should be increased to produce the total pressure gradient between the inlet and the outlet of the domain.

This suggest that the total pressure loss in normalized form Equation (3), may be established as the objective function.

$$pt_{loss} = 1 - \frac{pt}{pt_{ref}}, \tag{3}$$

Where the reference total pressure pt_{ref} corresponds to the inlet pressure condition, and the total pressure pt at the outlet of the seal is defined in Equation (4).

$$pt = p\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} \tag{4}$$

Where γ is a ratio of specific heats $\gamma = 1.4$, and M is Mach number. In this study, FLUENT software calculates the pressure fields at the domain's exit boundary, and MATLAB processes this information to obtain the final value of the objective function, points 5, 6 and 7 (**Figure 4**).

RESULTS

This section presents the final shape reached of the labyrinth as result of the optimization process.

Numerical Design Optimization

The minimization of the objective function for each seal configuration was executed by the optimization process and the runs are depicted in (**Figure 11**). It presents the variation of the outlet pressure throughout the entire process. The schematic representation of the optimization processes illustrates how the robust algorithm was able to reach a practical solution without find no local minimal.

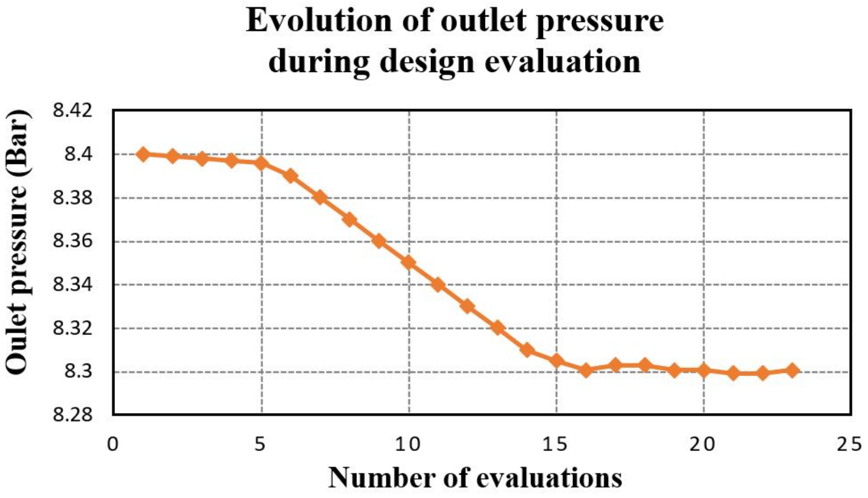


Figure 11. Minimization of the objective function using Genetic Algorithms (GA) Behaviour of the outlet pressure along the optimization process.

The behaviour of the design variables *x* and *m* throughout the optimization process is illustrated in (**Figure 12**). While the seal radius *m* was kept constant during the process, *x* experienced an extreme variation, starting from a straight seal with coordinate $x = -0.001$ and reaching a final value of $x = -2.5$. An extreme value of this parameter could cause the seal to collide with the top wall.

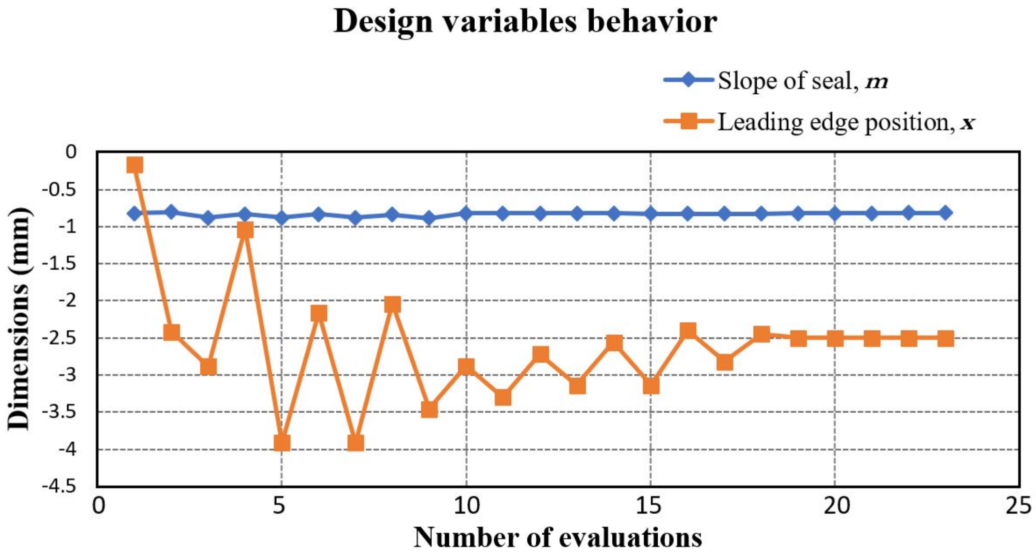


Figure 12. Behaviour of design variables *m* and *x*, across 23 automated evaluations.

Final shape of the seal.

The optimization process reached the best shape seal, (**Figure 13**), for the leading edge which reaches an inclination of 45° and a magnitude of -2.5 *mm* for the variable x as shown in (**Figure 6**).

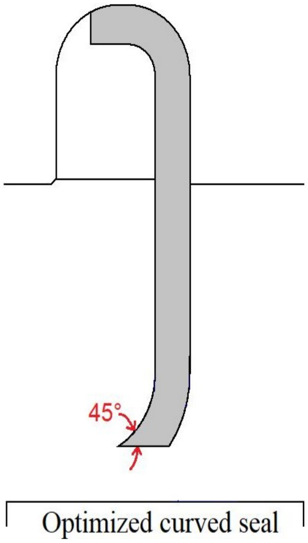


Figure 13. Optimized geometry shape of the labyrinth seal.

The 3D CAD seal model of the best geometry generated during the optimization process is presented in (**Figure 14**). In the sectional view, the shape that defines the leading edge and the geometric characteristics of the new inclined geometry are shown.

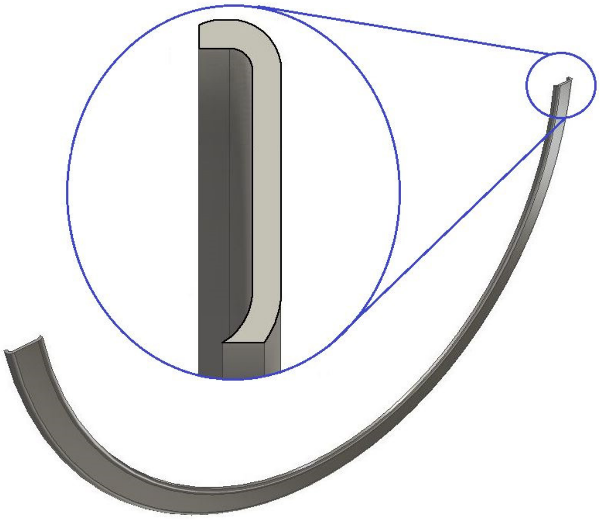
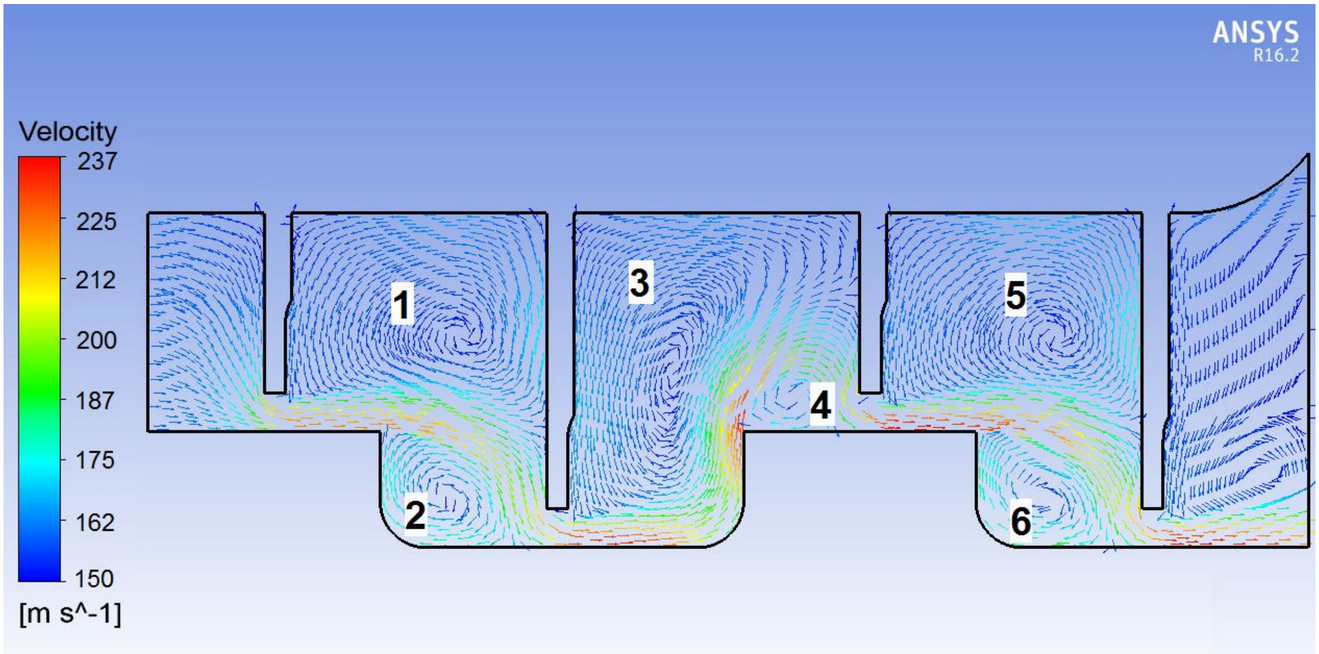


Figure 14. 3D CAD model of the seal obtained by the optimization process.

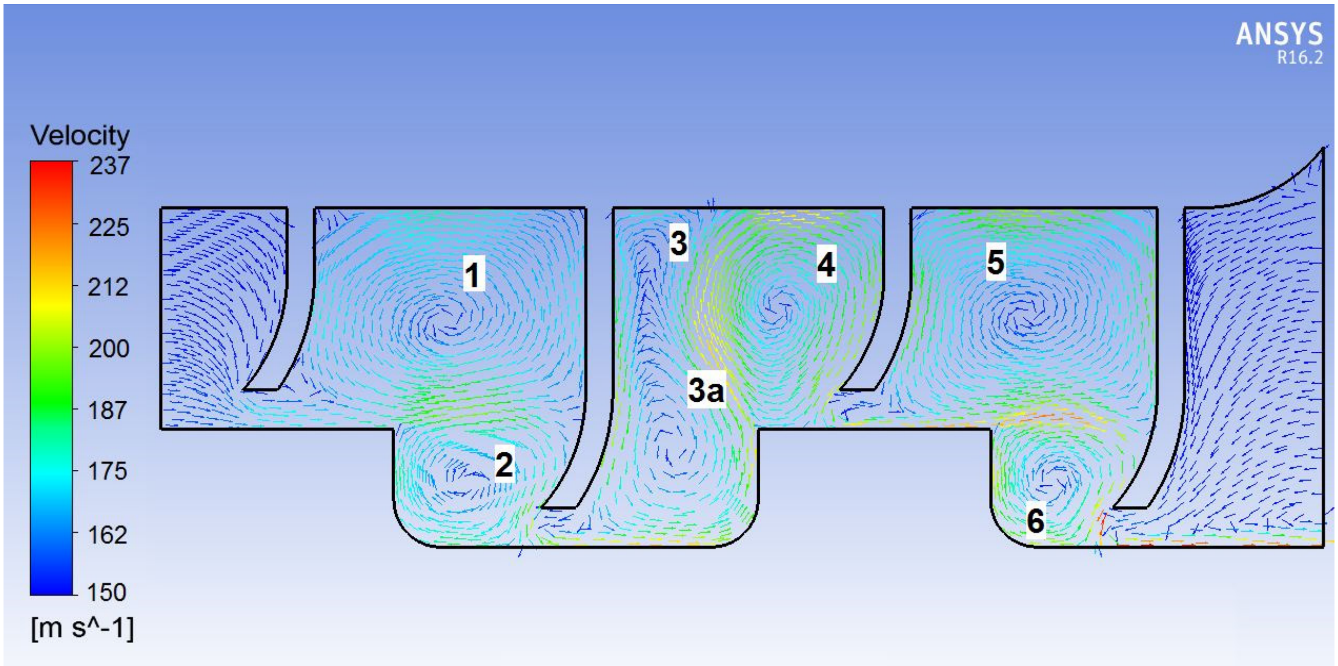
Flow behaviour

In this section, using CFD, the flow is analysed throughout the computational domain, comparing the original straight seal against the new seal with the curved geometry.

The velocity field comparison between both geometries with different shapes is shown in (**Figure 15**). For the original seal (a), the simulation yields six vortices. However, in chamber two, the optimized curved seal (b) exhibits vortex bipartition and growth of the fourth vortex, allowing the steam to follow a longer flow path from the gap.



(a) Original straight seal.

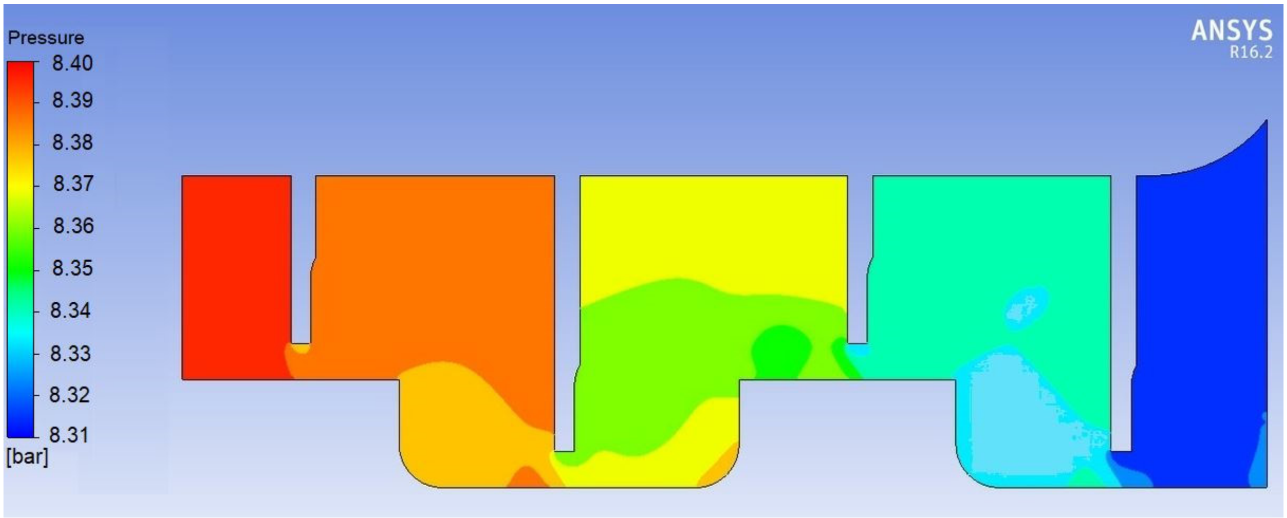


(b) Optimized curved seal.

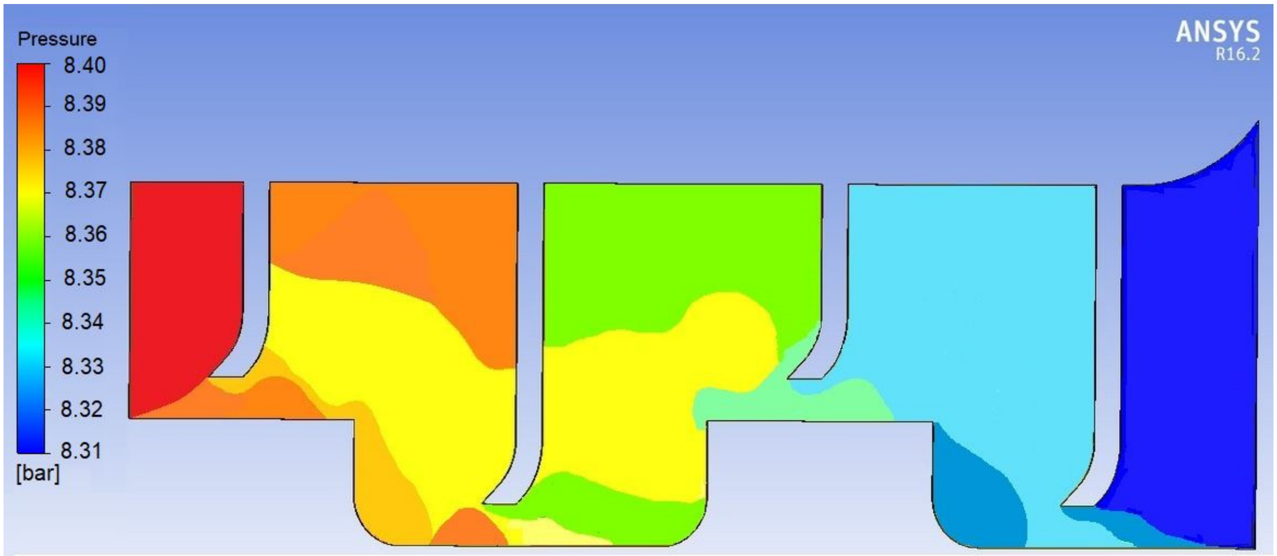
Figure 15. Comparison of the fluid trajectory inside chamber cavities of the labyrinth seal.

Static Pressure

The analysis reveals a reduction in static pressure intensity across all seal chambers. In the optimized seal design, a notable expansion of low-pressure zones is observed, indicating improved performance. This trend is illustrated in (**Figure 16**), which compares the pressure distribution between configurations.



(a) Original straight seal.



(b) Optimized curved seal.

Figure 16. Comparison between the original and optimized seal of the pressure ratio decrease along the chamber cavities.

DISCUSSION

The utilization of the algorithm helped to reduce the outlet pressure in the seal system by 1.2%, using only 23 evaluations in the optimization process which, for a total of 5 stages in the turbine with the same seals systems, would result in a 6% pressure reduction. Spite the pressure ratio decreased, the seal geometry, although inclined, is feasible to manufacture with current technology.

Additionally, the optimized seal reduces the flow velocity magnitude through the gaps of the first and second chambers. The velocity of the steam leaving this chamber results in an increase in turbulence intensity, particularly in vortex 3a. The steam jet follows a longer trajectory and, as a consequence of wall friction, undergoes a velocity reduction, which contributes to an increase in the pressure gradient. The performance of smoke along the seal model is similar to that described by Darshan P (9), who noticed that two vortexes were created in each chamber of the seal.

Zhang *et al.* (24) improved the efficiency of the seal system, their proposal yields a reduction in the leakage flow rate of 42.7~43.6%. However, manufacturing the proposed geometric shapes with current commercial technology would be complicated.

Kim and Cha (14) increased the number of seals to reduce the pressure ratio with the staggering method. Cases were analysed that employed a leading-edge configuration but utilized regular geometries for the seal tip, similar to the study conducted by Dogu *et al.* (7), since seal designs, such as the mushroom type, are challenging to manufacture.

The analysis of the flow parameters of the redesign seal confirms that using the coupling of different software controlled by an optimization algorithm could be established as a straightforward methodology to reach the ideal geometric seal profile and above all, with realistic manufacturing possibilities.

CONCLUSIONS

This research presented the numerical optimization of a new seal shape given by a 2D asymmetrical CFD model as a fast evaluator of the flow within the domain of interest.

Comparison between straight and inclined seals showed a significant change in the steam's flow behaviour along the back pressure chambers. Increasing the number of vortices was fundamental to achieving a pressure reduction. Another critical factor was the permanence of the steam in the chambers; the longer the steam travels, the more pressure in the chamber decreases.

Despite its simplicity, it is recommended improving the numerical model to make it more robust and computationally accessible to increase the number of process evaluations. Besides, having a valid reference of the flow behaviour inside the seals, the optimized shape could analyse the ideal flow characteristics inside the chamber by creating a high-fidelity numerical model since there is no experimental data.

Finally, the methodology presented in this work could contribute to re-powering turbines by designing new shapes or applying them to other forms of seals, making the energy transfer in each turbine step more efficient.

FUNDING

To CONAHCYT of México.

DECLARATION OF COMPETING INTEREST

The authors have declared no conflict of interest

ACKNOWLEDGMENTS

To CONAHCYT of México, for the financial support to the first author. This paper has been done under the framework of the Aulas CIMNE, Morelia project.

ETICAL CONSIDERATIONS

This research has been developed under the standars of the Comité de Ética en Investigación, Investigación y Bioseguridad of the Universidad Michoacana de San Nicolas de Hidalgo.

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