Integration of Virtual Manipulatives for Teaching and Learning Perimeter and Area in Lower Secondary Education

Integración de Manipulativos Virtuales para la Enseñanza y el aprendizaje del Perímetro y el Área en la Educación Básica Secundaria

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Highlights

- Virtual manipulatives significantly improved students' understanding of perimeter and area.
- Students in the experimental group showed greater motivation, engagement, and conceptual clarity.
- The integration of interactive digital tools supports both cognitive and affective dimensions of mathematics learning.

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ABSTRACT

Introduction. The integration of digital resources in mathematics education has shown promise in improving students' conceptual understanding. Virtual manipulatives, in particular, are designed to support abstract reasoning through interactive and visual learning environments. **Objective.** This study aimed to evaluate the impact of virtual manipulatives on seventh-grade students' learning outcomes and engagement during the instruction of perimeter and area. **Materials and Methods.** A quasi-experimental, mixed-methods design was employed involving two intact seventh-grade classes (N=32) from a public school in Mitrovica, Kosovo. One class served as the experimental group, receiving instruction with virtual manipulatives, while the control group followed traditional methods. The intervention spanned two weeks and included pre- and post-tests, a student questionnaire, and an observation checklist. Quantitative data were analyzed using SPSS software, including paired and independent samples t-tests, while qualitative insights were gathered through observation. **Results.** The experimental group demonstrated significantly higher post-test scores compared to the control group ($p = .005$), indicating improved understanding of perimeter and area. Questionnaire results revealed high levels of motivation, enjoyment, and perceived learning benefits among students exposed to virtual manipulatives. Observation data supported these findings, showing increased engagement, collaboration, and positive behavioral changes. **Conclusions.** The findings confirm that virtual manipulatives positively influence students' academic performance and attitudes in mathematics. They support both cognitive and affective domains of learning, offering an effective strategy for teaching geometric concepts. Further research is encouraged to explore long-term effects across broader student populations.

RESUMEN

Introducción. La incorporación de recursos digitales en la enseñanza de las matemáticas ha mostrado beneficios en la comprensión conceptual de los estudiantes. Los manipulativos virtuales, en particular, facilitan el razonamiento abstracto mediante entornos de aprendizaje interactivos y visuales. **Objetivo.** Este estudio tuvo como propósito evaluar el impacto del uso de manipulativos virtuales en los resultados de aprendizaje y en el compromiso de estudiantes de séptimo grado al abordar los conceptos de perímetro y área. **Materiales y métodos.** Se utilizó un diseño cuasi-experimental con enfoque mixto, involucrando a dos clases de séptimo grado (N=32) de una escuela pública en Mitrovica, Kosovo. Un grupo experimental recibió instrucción con manipulativos virtuales, mientras que el grupo control siguió métodos tradicionales. La intervención duró dos semanas e incluyó pruebas pre y post, un cuestionario estudiantil y una lista de observación. Los datos cuantitativos se analizaron con SPSS mediante pruebas t pareadas e independientes, complementados con observaciones cualitativas. **Resultados.** El grupo experimental obtuvo puntuaciones significativamente más altas en la prueba post-intervención ($p = .005$), lo que evidencia una mejor comprensión de los conceptos. Los cuestionarios reflejaron altos niveles de motivación, disfrute y percepción de aprendizaje. Las observaciones indicaron una mayor participación, colaboración y cambios de comportamiento positivos. **Conclusiones.** Los manipulativos virtuales demostraron ser eficaces para mejorar el rendimiento académico y las actitudes hacia las matemáticas, favoreciendo tanto los procesos cognitivos como afectivos. Se recomienda continuar investigando sus efectos a largo plazo en poblaciones estudiantiles más amplias.

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INTRODUCTION

Mathematics is a discipline that demands the acquisition of abstract concepts, a process that poses significant challenges for many students. This difficulty often stems from the need to construct internal mental models of ideas that are not immediately accessible through sensory experience. Such challenges are particularly evident in geometry, in understanding concepts such as size and spatial relationships, and in tackling problems that require project-based thinking or interpreting symbolic representations of physical objects.

Among the specific difficulties observed in mathematics education, problem-solving related to perimeter and area stands out. Many students confuse these two concepts, struggling to differentiate between the measurement of a figure's boundary and the surface it encloses. The issue goes beyond the mere application of formulas; it often reflects a lack of fundamental understanding of the concepts being measured. This misunderstanding is frequently attributed to teaching approaches that emphasize procedural knowledge over concrete or visual experiences [(1)]. Additionally, when confronted with tasks involving textual descriptions, students may fail to mentally visualize the figures described, leading to difficulties in identifying relevant data and selecting appropriate problem-solving strategies. They tend to memorize formulas mechanically, yet struggle to transfer their knowledge to novel contexts—particularly those involving composite figures or real-life applications.

Another common obstacle involves dealing with different units of measurement. Students frequently confuse units or overlook the need for conversions, which results in incorrect solutions. These difficulties are symptomatic of a broader disconnect between abstract mathematical concepts and students' lived experiences. Without early intervention to bridge these gaps, students' learning continuity and motivation can suffer [(2)]. Given the diversity of learning styles within any classroom, it is essential that teachers employ instructional strategies that help students construct new understandings based on their prior knowledge. Neglecting these differences risks rendering students passive and disengaged, ultimately limiting their academic growth.

To address challenges in abstract reasoning, educators have increasingly turned to manipulatives—both physical and virtual—as instructional tools [(3)]. These manipulatives support students in developing concrete understandings of mathematical ideas and in making meaningful connections between objects, symbols, and abstract representations [(4)]. Their use has also been associated with increased student engagement and participation, which can enhance the overall effectiveness of the learning process [(5)]. Virtual manipulatives, in particular, are defined as "online visual and interactive representations of dynamic objects that provide opportunities for building mathematical knowledge" [(6)]. Unlike static models, which merely depict physical tools, virtual manipulatives enable direct interaction, a critical feature for fostering abstract thinking [(7)]. These tools also allow educators to tailor instruction to diverse learning needs and offer inclusive strategies for students with learning difficulties [(8)].

The flexibility of virtual tools is another notable advantage: they are widely accessible, cost-effective, and do not require physical storage or classroom space. They permit manipulations—such as resizing or recoloring geometric shapes—that are not feasible with physical materials, providing students with enriched opportunities to explore and understand mathematical concepts [9]. Furthermore, when contextualized within real-world applications, these tools can bridge the gap between theory and practice, equipping students with transferable skills for daily life [10].

Recent international studies have provided extensive evidence on the impact of manipulatives on mathematics learning [11-14]. A meta-analysis [15] concludes that virtual manipulatives yield superior outcomes compared to traditional instructional methods, with positive effects observed across grade levels, study durations, and student populations. Empirical data show that consistent use of virtual manipulatives enhances students' understanding of concepts such as area and perimeter. These tools also foster multiple problem-solving strategies, critical thinking, and logical reasoning [4]. In both physical and digital formats, manipulatives have been shown to contribute significantly to student achievement in mathematics [16-17].

Virtual tools offer substantial potential for integration into instructional design, enabling teachers to implement continuous and personalized learning experiences [18]. These tools are particularly effective when deliberately incorporated into pedagogical planning, as they can enhance students' conceptual understanding of mathematical ideas. According to Clements [12], manipulatives are beneficial because they align with students' cognitive structures, are easily understood, offer flexible use, and help build conceptual bridges. However, their effectiveness hinges on the teacher's ability to guide students in interpreting mathematical representations and drawing connections between concepts, thereby imbuing manipulatives with genuine educational value [14,19].

Importantly, virtual manipulatives have proven to be a sustainable and accessible technology for supporting students with learning disabilities in mastering complex mathematical concepts [20]. Their interactive, visual, and dynamic features facilitate the construction of abstract meanings. For students with specific learning difficulties, these tools serve as a bridge between concrete experiences and symbolic reasoning, offering customizable features and opportunities for repeated practice—both of which are crucial for long-term retention and confidence building.

In this context, the integration of innovative digital technologies in primary mathematics instruction is strongly recommended. Interactive applications and visual platforms enhance learners' grasp of abstract concepts, increase motivation, and support individualized pacing [21]. Technology-driven instruction transforms the learning process into a more engaging, hands-on, and student-centered experience, ultimately preparing learners to meet the cognitive demands of the 21st century.

In sum, the strategic use of manipulatives—particularly virtual ones—emerges as a powerful means to overcome the inherent abstraction in mathematics. These tools can enhance student engagement, deepen conceptual understanding, and foster lasting improvements in mathematical performance.

MATERIALS AND METHODS

This research follows the principles of action research conducted in a classroom setting. The design adopts a mixed-methods, quasi-experimental approach involving two seventh-grade classes from Fazli Grajçevci Primary School in Mitrovica, Kosovo—one designated as the experimental group and the other as the control group, each comprising 16 students. The primary objective was to assess the effectiveness of virtual manipulatives in teaching the mathematical concepts of perimeter and area. The study was conducted over a two-week period, from December 2024.

In the experimental group, instruction focused on the integration of virtual manipulatives, while the control group received traditional instruction based on standard classroom practices. Both groups participated in a pre-test and post-test. In addition, data were collected from the experimental group using an observation checklist and a student questionnaire.

The research instruments were validated prior to implementation. Validation involved expert consultations in the field of mathematics education and a detailed content analysis to confirm alignment with the targeted mathematical constructs. Furthermore, the instruments were piloted with a small group of students who shared characteristics with the study participants, allowing for the identification and correction of ambiguities, thus ensuring clarity and functionality during the main study.

Since the aim of our research is to investigate the impact of virtual manipulatives on students' understanding of perimeter and area, the following research questions were formulated based on this objective: i)What is the impact of using virtual concrete tools on learning the concept of perimeter and area among seventh-grade students?;This was supplemented by two sub-questions: i) What is the increase or decrease in students' performance before and after using virtual concrete tools?; ii) What are students' perceptions and changes in behavior and attitudes when using virtual concrete tools?.

The research procedure adhered to a structured sequence. The first step involved administering the pre-test to both classes to establish a baseline understanding of perimeter and area. Subsequently, the instructional phase began: in the experimental group, teaching utilized virtual manipulatives, while in the control group, instruction was delivered using traditional methods. After the instructional phase, both groups completed a post-test to compare learning outcomes. The final data collection activities involved administering the questionnaire and observation checklist in the experimental group.

The intervention took place across eight instructional sessions over the course of two weeks—four sessions each week. After obtaining authorization from the school administration, the two study groups were selected. On the first day, both classes engaged in introductory activities to assess prior knowledge related to perimeter and area. On the second day, a shared instructional session ensured that all students had a common baseline understanding of the key concepts.

On the third day, both classes completed a pre-test designed to measure their initial grasp of perimeter and area. The intervention began on day four and continued through day seven. During this phase, the control group received instruction using conventional textbook-based methods, while the experimental group engaged in lessons enhanced by virtual manipulatives, with the goal of facilitating conceptual development and reducing cognitive load.

Digital tools used in the intervention included Math Playground (https://www.mathplayground.com), specifically the "Hidden Math Concepts" section. The Area Blocks game was utilized to support students' exploration of perimeter and area through interactive experiences.

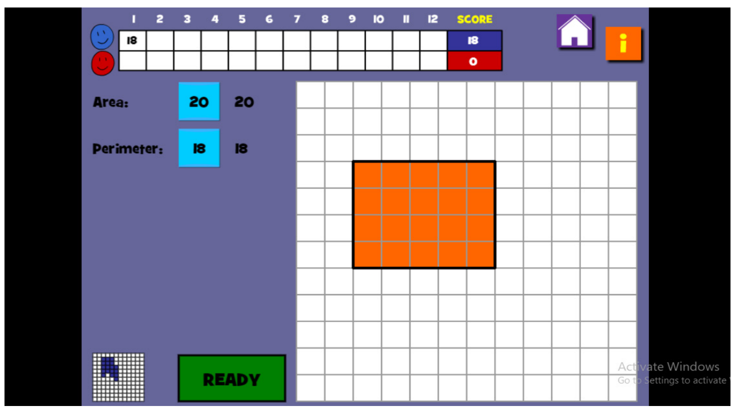

Figure 1. Virtual manipulative for surface design

Students were guided through problem-solving tasks in a virtual environment. For example, one task required constructing a figure with an area of 20 square units and a perimeter of 18 linear units. Each student worked independently to develop a solution, while peers observed and contributed alternative strategies. These activities supported collaborative learning and encouraged flexible spatial reasoning.

The intervention continued with the introduction of Toy Theater (http://www.toytheater.com), which offers interactive games in subjects such as mathematics, literacy, and the arts. Within the Virtual Manipulatives > Geometry section, the "Area Perimeter Explorer" tool was employed.

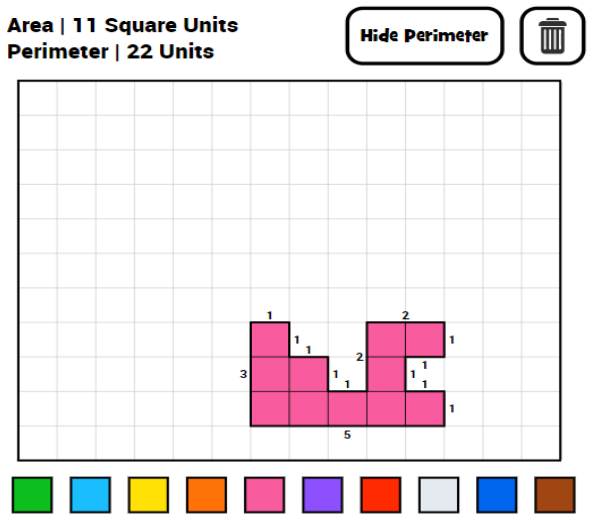

Figure 2. Virtual manipulative for exploring perimeter and area

This tool allowed students to construct shapes by adding or removing unit squares. Real-time feedback on area and perimeter changes provided visual scaffolding, enabling students to better internalize mathematical relationships and reducing cognitive overload.

Subsequently, more advanced games were introduced, such as Perimeter Climber (http://www.toytheater.com/perimeter-climber/). In this game, students are presented with four rectangles with known side lengths and are asked to identify the one with a specific perimeter, e.g., 12 units.

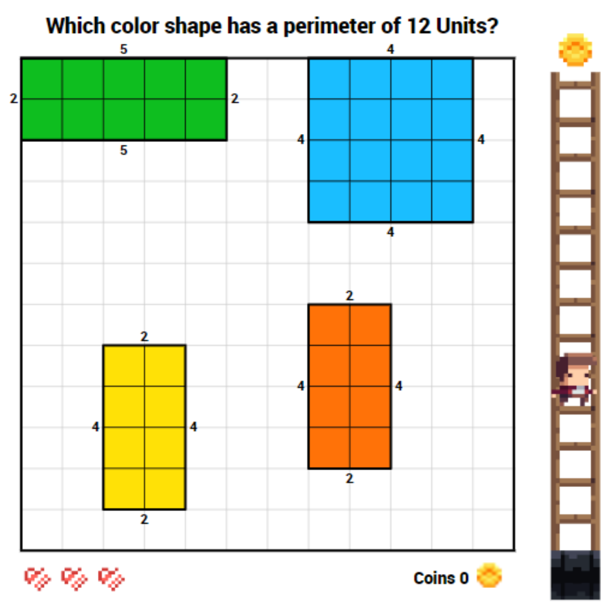

Figure 3. Virtual perimeter game

If the student selects the correct shape, they "climb" the ladder and earn a virtual coin; incorrect answers cause them to fall back to the start. Each student had three attempts to complete the challenge. The game allowed them to explore various side-length combinations that result in the same perimeter, enhancing conceptual flexibility.

Another interactive game introduced was Area Climber (http://www.toytheater.com/area-climber/), where students were tasked with selecting the rectangle that matched a given area.

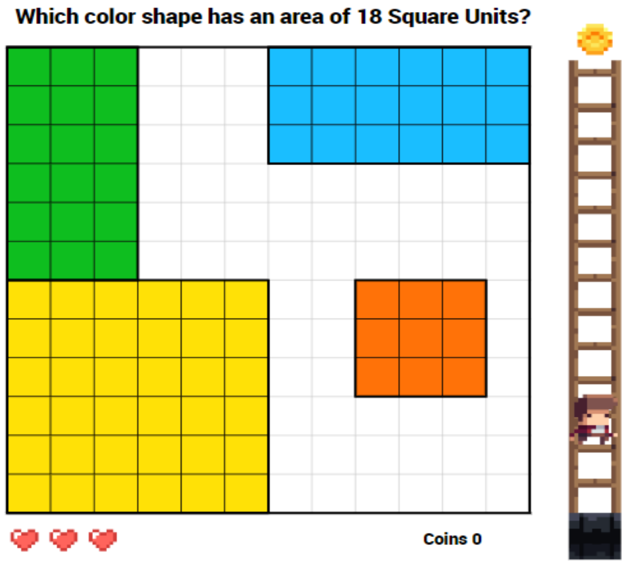

Figure 4. Virtual game for the surface

Throughout the intervention, students' participation was actively monitored using observation checklists. Teachers recorded behavioral notes and used a points system to motivate engagement. Students were also encouraged to use the virtual manipulatives at home to reinforce classroom instruction. The intervention concluded on day seven. On day eight, all students completed the post-test to evaluate conceptual gains and compare the effectiveness of virtual and traditional instructional strategies.

Data analysis was conducted using SPSS software. Independent samples t-tests and paired samples t-tests were employed to compare pre-test and post-test results within and between groups. Descriptive statistics, including means and standard deviations, were calculated to evaluate student performance and the effectiveness of the virtual tools. Analysis and interpretation of results were aligned with the research questions, and the final section of the study presents recommendations based on the findings.

The full version of the questionnaire and observation checklist used in this study are available as supplementary material at Arifi, Arber; Mahmuti, Agon (2025), "Appendix", Mendeley Data, V1, doi: 10.17632/56mr7hsh3f.1.

RESULTS

Questionnaire Data Analysis and Results

A total of 16 students from the experimental group completed the post-intervention questionnaire. The instrument consisted of eight closed-ended items rated on a five-point Likert scale ranging from 1 (*strongly disagree*) to 5 (*strongly agree*). Tables 1 and 2 present the descriptive statistics for all items.

Students showed a strong liking for mathematics (Item 1) and reported high levels of enjoyment when learning about perimeter and area through computer-based methods (Item 3). They also acknowledged the educational utility of digital games (Item 4). Conversely, students expressed disagreement with the notion that learning solely from textbooks is effective (Item 2) (**Table 1**).

Table 1. Descriptive statistics for student responses on perception of mathematics and the use of virtual manipulatives.

Item	Mean	Median	Mode	SD	Min	Max
1. I like math	5.00	5.00	5.00	0.000	5.0	5.0
2. It's easy to learn perimeter and area just from a book	2.44	2.00	2.00	1.153	1.0	5.0
3. I enjoyed learning perimeter and area on the computer	4.94	5.00	5.00	0.250	4.0	5.0
4. The computer game helped me learn about perimeter and area	4.44	5.00	5.00	0.892	3.0	5.0

n = 16. No missing data reported; **SD** = Standard Deviation

This positive perception was further reinforced by students' feedback on the game-based activities, which they found clear and manageable (Item 5), while also supporting the inclusion of more digital games in the classroom (Item 6). Additionally, they noted that such tools increased their appreciation for mathematics (Item 7) and made the post-test feel easier (Item 8) (**Table 2**).

Table 2. Descriptive statistics for student responses regarding difficulty and motivation

Item	Mean	Median	Mode	SD	Min	Max
5. The perimeter and surface play has been difficult	1.06	1.00	1.00	0.25	1.0	2.0
6. I wish we had more games like this on computers	5.00	5.00	5.00	0.00	5.0	5.0
7. Learning and playing on the computer has made me like math more	5.00	5.00	5.00	0.00	5.0	5.0
8. The test seemed easier to me after playing computer games	4.88	5.00	5.00	0.34	4.0	5.0

n = 16. No missing data reported; **SD** = Standard Deviation

Observation Data and Results

To further triangulate the results, observational data were gathered during classroom sessions in the experimental group. The observation checklist consisted of six closed-ended items rated with "Yes" or "No" (**Table 3**).

Most students showed high levels of concentration and interest during lessons involving virtual manipulatives. All students completed their tasks, and 15 out of 16 demonstrated collaborative behavior and increased activity during and after using digital tools. A small proportion (25%) did not engage in board-based tasks, though this did not impact their general motivation or participation.

Table 3. Summary of observation checklist results in the experimental group (n = 16)

Observation	Yes (n)	Yes (%)	No (n)	No (%)
1. Student is focused during the learning unit	16	100%	0	0%
2. Student is committed to solving tasks on the board	12	75%	4	25%
3. Student shows interest during lessons using virtual tools	16	100%	0	0%
4. Student completes tasks while manipulating virtual tools	16	100%	0	0%
5. Student feels satisfaction and motivation while using virtual manipulatives	16	100%	0	0%
6. Student collaborates and increases activity during and after using virtual manipulatives	15	93.8%	1	6.3%

These behavioral findings reinforce the positive self-reported attitudes from the questionnaire and align with the research hypothesis regarding increased student motivation and engagement through the use of virtual tools.

Pre-Test and Post-Test Data Analysis

The analysis began by comparing the pre-test and post-test scores of students in both the control and experimental groups, providing initial insights into the effectiveness of the intervention through descriptive statistics for both phases of the study (**Table 4**).

At baseline, students in the control group had a mean score of 30.63 (SD = 7.04), while those in the experimental group had a slightly higher mean of 30.94 (SD = 6.12). An independent samples t-test revealed no statistically significant difference between the two groups prior to the intervention ($p = .894$), confirming that both groups started at a similar level of understanding.

Following the intervention, the post-test results showed a clear improvement in both groups. The control group achieved a mean score of 43.44, while the experimental group scored significantly higher, with a mean of 49.69. This 6.25-point difference was statistically significant ($p = .005$), suggesting that the instructional approach used in the experimental group—based on virtual manipulatives—was more effective in enhancing students' understanding of perimeter and area.

Table 4. Comparison of pre-test and post-test results in control and experimental groups

Group	Test Type	Mean	N	SD	SE
Control class	Pre-test	30.63	16	7.04	1.76
Control class	Post-test	43.44	16	5.39	1.35
Experimental class	Pre-test	30.94	16	6.12	1.53
Experimental class	Post-test	49.69	16	6.18	1.55

SD = Standard Deviation; **SE** = Standard Error

Independent samples t-tests were conducted to assess differences between the control and experimental groups. In the pre-test, the assumption of equal variances was met (Levene's test $F = 0.369$, $p = .548$), and no significant difference was found between group means ($t\,(30) = -0.134$, $p = .894$). In the post-test, Levene's test again confirmed equal variances ($F = 0.001$, $p = .972$), and a statistically significant difference was found between groups in favor of the experimental group ($t\,(30) = -3.048$, $p = .005$, 95% CI [–10.44, –2.06]).

Paired t-tests (**Table 5**) further confirmed the improvement within each group. Both groups showed statistically significant increases from pre-test to post-test. However, the experimental group exhibited a greater gain in performance (⊿M = 18.75) compared to the control group (⊿M = 12.81), reinforcing the effectiveness of virtual manipulatives in promoting deeper conceptual understanding.

Table 5. Paired t-test results (pre-test vs post-test) in control and experimental groups

Group	Δ *Mean*	SD	SE	t	df	p
Control class	12.81	7.06	1.77	–7.255	15	.000
Experimental class	18.75	7.19	1.80	–10.434	15	.000

SD = Standard Deviation; **SE** = Standard Error

DISCUSSION

The findings of this study indicate that virtual manipulatives can effectively support students' learning of mathematical concepts such as perimeter and area. Students in the experimental group demonstrated greater gains in post-test performance and reported higher levels of motivation and engagement compared to the control group. These results are consistent with earlier studies that highlight the value of concrete and digital tools in enhancing mathematical understanding and achievement [11, 15, 17].

Classroom observations further revealed notable behavioral changes among students. Learners who had previously shown limited participation became more involved and confident when interacting with virtual tools, a finding that aligns with Sutton and Krueger's [5] work on the motivational impact of digital manipulatives. This increase in engagement suggests that such tools can foster not only academic improvement but also positive shifts in student attitudes.

Moreover, the visual and interactive nature of virtual manipulatives allowed students to explore abstract mathematical relationships in a more concrete and accessible way. This supports the constructivist learning theory, which emphasizes active student involvement and the use of meaningful representations to construct knowledge [4,9,12].

Recent research extends and supports these findings. Altiparmak and Ercan [27] found that 4th-grade students taught with virtual manipulatives in an online setting showed significantly higher achievement and motivation compared to traditional instruction. Similarly, Ahmad and Siller [28] demonstrated that blending physical and virtual manipulatives in 5th graders produced substantial learning gains across all performance levels. These studies provide further empirical evidence of the pedagogical value of virtual tools across varying contexts and age groups.

Beyond mathematics content, students developed transversal skills such as communication, critical thinking, and collaborative problem solving. These outcomes support Wenglinsky's [22] findings on the broader educational benefits of active learning environments.

However, the study's scope is limited. The short duration of the intervention and the small sample size ($N = 32$) constrain the extent to which the findings can be generalized. Future research should investigate the long-term impact of virtual manipulatives, compare them with physical manipulatives across different mathematical topics, and examine their effects in more diverse educational settings. Additional studies could also explore their role in supporting students with learning difficulties [8].

CONCLUSIONS

This study suggests that virtual manipulatives are a promising instructional resource for improving students' understanding of perimeter and area. Learners exposed to digital tools demonstrated better academic performance and higher engagement compared to those receiving traditional instruction. While these results are encouraging, they should be interpreted as preliminary due to the limited sample size and short duration of the intervention. Nonetheless, virtual manipulatives offer practical benefits that merit further exploration and integration into mathematics education.

ETHICAL CONSIDERATIONS

This research followed recognized ethical standards for studies involving human participants. Informed consent was obtained, and students were informed of their right to withdraw at any time without consequences. Anonymity and data confidentiality were strictly maintained throughout.

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DECLARATION OF COMPETING INTEREST

The authors have declared no conflict of interest.

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